(1) (a) We want to solve

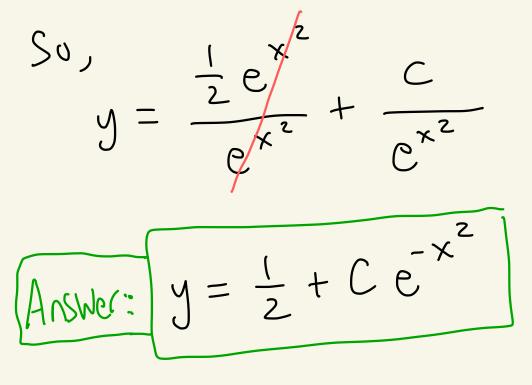
$$y' - zy = 1$$

on $I = (-\infty,\infty)$.
Let $A(x) = \int -2dx = -2x$
Multiply by $e^{A(x)} = e^{-2x}$ to get
 $e^{-2x}y' - 2e^{-2x}y = e^{-2x}$
This gives
 $(e^{-2x}y)' = e^{-2x}$
Let $A(x) = \int e^{-2x}y = e^{-2x}$
This gives
 $(e^{-2x}y)' = e^{-2x}$
Let $A(x) = \int e^{-2x}y = e^{-2x}$
 $(e^{A(x)}y)'$
Integrating with respect to x gives
 $e^{-2x}y = \int e^{-2x}dx$
So, $e^{-2x}y = -\frac{1}{2}e^{-2x} + C$
Then, $y = -\frac{1}{2}e^{-2x} + \frac{C}{e^{-2x}} = e^{2x}$
Answer: $y = -\frac{1}{2} + Ce^{2x}$

D(b)
We want to solve

$$y' + 2 \times y = x$$

on $I = (-\infty,\infty)$
Let $A(x) = \int 2x \, dx = x^2$
Multiply by $e^{A(x)} = e^{x^2}$ to get
 $e^{x^2}y' + 2xe^{x^2}y = xe^{x^2}$
So, $(e^{x^2}y)' = xe^{x^2}$ ($e^{A(x)}y$)'
Thus, by integrating with respect to x we get
 $e^{x^2} \cdot y = \int xe^{x^2} dx$
We have that
 $\int xe^{x^2} dx = \int \frac{1}{2}e^{x} \, du = \frac{1}{2}e^{x} + c = \frac{1}{2}e^{x} + c$
Thus,
 $e^{x^2} \cdot y = \frac{1}{2}e^{x^2} + c$



() (c) We want to solve $\frac{dy}{dx} + 2xy = xe^{-x^2}$ $vn I = (-\infty, \infty)$ Let $A(x) = \int 2x \, dx = x^2$ Multiply hoth sides by $e^{A(x)} = e^{x}$ to get e^{χ^2} , $\frac{dy}{d\chi}$ + e^{χ^2} . $Z\chi y = e^{\chi} \chi e^{\chi^2}$ $\frac{1}{2} + \frac{1}{2} + \frac{1}$ Thus, $(e^{x}, y) = x$ $e^{2}e^{-x^{2}} = \frac{e^{x^{2}}}{e^{x^{2}}} = 1$ Integrating with respect to x gives: this is always $e^{x} \cdot y = \int x \, dx$ $\left(\begin{array}{c} e^{A(x)} \\ \end{array} \right)'$ $e^{x} \cdot y = \frac{x}{2} + C$ Thus, $y = \frac{x^2}{e^{x^2}} + \frac{c}{e^{x^2}}$ Answer $y = \frac{1}{2}x^2e^{-x^2} + Ce^{-x^2}$

$$\begin{array}{l} \textcircledlet\\ y' + \frac{1}{x}y = 3x^2 - \frac{1}{x} \\ on \ T = (o, \infty). \\ Let\\ A(x) = \int \frac{1}{x} dx = ln|x| = ln(x) \\ A(x) = \int \frac{1}{x} dx = ln|x| = ln(x) \\ f = e^{n(x)} = x \\ e^{A(x)} = e^{n(x)} = x \\ e^{A(x)} = e^{n(x)} = x \\ of \ y' + \frac{1}{x}y = 3x^2 - \frac{1}{x} \\ e^{n(B)} = B \\ by \ x \ to \ get: \\ x \ y' + y = 3x^3 - l \\ This \ becomes \\ (x \ y)' = 3x^3 - l \\ (e^{A(x)} \ y') \\ (e^{A(x)} \ y') \\ \end{array}$$

Thus, by integrating both sides with
respect to x we get
$$xy = \int (3x^3 - 1) dx$$

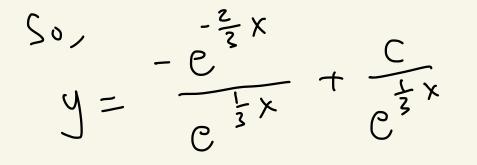
$$\begin{aligned} So, \\ xy &= \frac{3}{4}x - x + C \end{aligned}$$

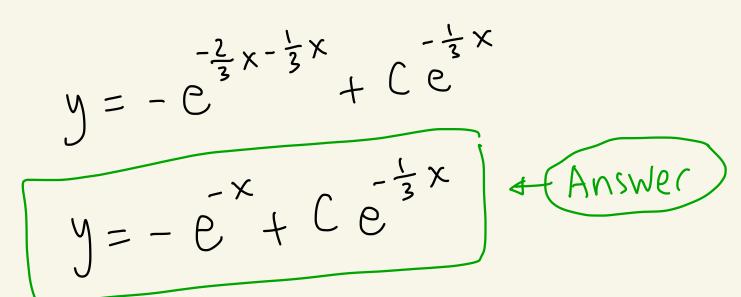
Thus,

$$y = \frac{3}{4}x^{3} - 1 + \frac{c}{x}$$
 Answer

2(a) We want to solve $3\frac{dy}{dx}+y=2e^{-x}$ Divide by 3 to put the ODE into standardized form. We get $\frac{dy}{dx} + \frac{1}{3}y = \frac{2}{3}e^{-x}$ Let $A(x) = \int \frac{1}{2} dx = \frac{1}{2}x$. Multiply both sides by $e^{A(x)} = e^{\frac{1}{3}x}$ to get $e^{\frac{1}{3}x} \cdot \frac{1}{3}y + \frac{1}{3}e^{\frac{1}{3}x}y = \frac{1}{3}e^{\frac{1}{3}x} - x$ $e^{\frac{1}{3}x} \cdot e^{-x} = e^{\frac{1}{3}x - x} = e^{\frac{1}{3}x}$ this is always So, $(e^{\frac{1}{3}x}, y)' = \frac{2}{3}e^{-\frac{2}{3}x}$ $\left(e^{A(x)}y\right)'$ Thus, $e^{\frac{1}{3}x}$, $y = \int \frac{2}{3}e^{\frac{2}{3}x} dx$ $\int_{-\frac{3}{2}}^{-\frac{3}{2}} e^{\frac{3}{2}x} dx = \frac{-\frac{3}{2}}{-\frac{3}{2}} (-\frac{-\frac{3}{2}}{-\frac{3}{2}} e^{\frac{3}{2}x}) + C$ $= -e^{(z_{3})x} + c$

Thus,
$$e^{\frac{1}{3}x} \cdot y = -e^{-\frac{2}{3}x} + c$$





(2)(b) We want to solve

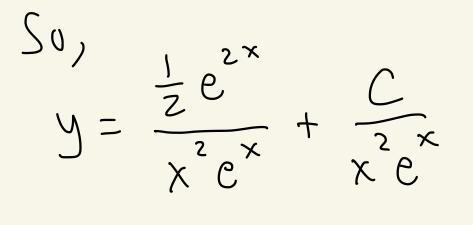
$$x^{2}y' + x(x+2)y = e^{x}$$

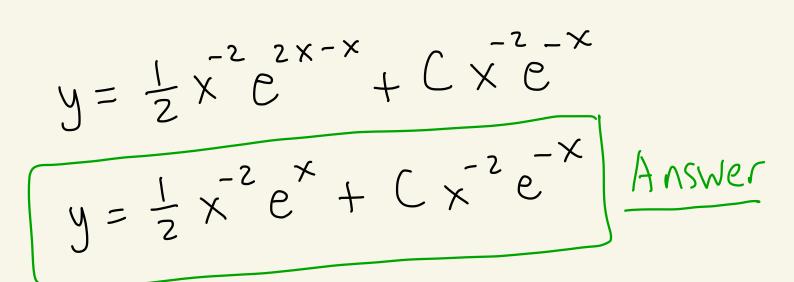
on $I = (0,\infty)$
First divide by x^{2} to put the UDE into
Standardized form. We get
 $y' + (1 + \frac{z}{x})y = x^{-2}e^{x}$
Let
 $A(x) = \int (1 + \frac{z}{x})dx = x + 2\ln|x| = x + 2\ln(x)$
We have
 $e^{A(x)} = e^{x+2\ln(x)} = e^{x}e^{2\ln(x)} = e^{x}e^{\ln(x^{2})} = x^{2}e^{x}$
So multiply both sides of $y' + (1 + \frac{z}{x})y = x^{2}e^{x}$
So multiply both sides of $y' + (1 + \frac{z}{x})y = x^{2}e^{x}$
This simplifies to
 $x^{2}e^{x}y' + (x^{2}+2x)e^{x}y = e^{2x}$

We get

$$(x^2e^xy)' = e^{2x}$$
 this is
always
Integrating with respect $(e^{A(x)}y)'$
to x gives

$$\chi^{2} \chi = \frac{1}{2}e^{2\chi} + C$$





$$(2)(c) We want to solve
$$(x^{2}+9)\frac{dy}{dx} + xy = 0$$
on $I = (-\infty,\infty)$
Divide by $x^{2}+9$ to put the ODE into
a standardized form. We get

$$\frac{dy}{dx} + \frac{x}{x^{2}+9} y = 0$$
Let $A(x) = \int \frac{x}{x^{2}+9} dx = \int \frac{1}{2} \frac{1}{2} u du = \frac{1}{2} \ln |u|$

$$u = x^{2}+9$$

$$u = x^{2}xdx$$

$$= \frac{1}{2} \ln (x^{2}+9)$$
Multiply both sides of the ODE by

$$Multiply both sides of the ODE by$$
Multiply both sides of the ODE by

$$e^{A(x)} = e^{\frac{1}{2}\ln(x^{2}+9)} \ln((x^{2}+9)^{1/2})$$

$$e^{A(x)} = e^{\frac{1}{2}\ln(x^{2}+9)} \ln((x^{2}+9)^{1/2})$$

$$e^{A(x)} = e^{\frac{1}{2}\ln(x^{2}+9)} \ln((x^{2}+9)^{1/2})$$

$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln((x^{2}+9)^{1/2})$$

$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln(x^{2}+9)^{1/2}$$

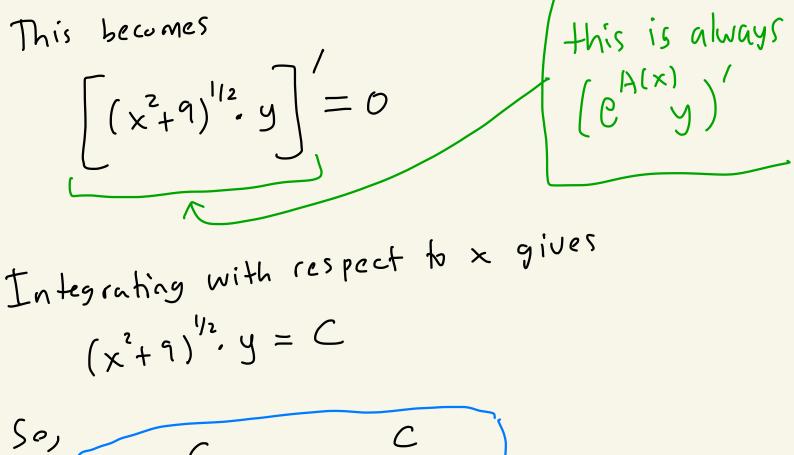
$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln(x^{2}+9)^{1/2}$$

$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln(x^{2}+9)^{1/2}$$

$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln(x^{2}+9)^{1/2}$$

$$f = e^{\frac{1}{2}\ln(x^{2}+9)} \ln(x^{2}+9)^{1/2}$$$$

$$(x^{2}+9)^{1/2}\frac{dy}{dx} + \frac{x}{(x^{2}+9)^{1/2}}y = 0$$



$$y = \frac{C}{(\chi^2 + 9)^{1/2}} = \frac{C}{\sqrt{\chi^2 + 9}}$$

3 We want to solve

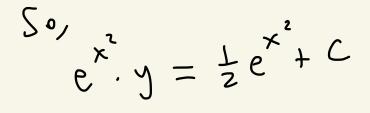
$$\frac{dy}{dx} + 2xy = x, \quad y(o) = -3$$
on $I = (-\infty, \infty)$
First we must find the general solution to

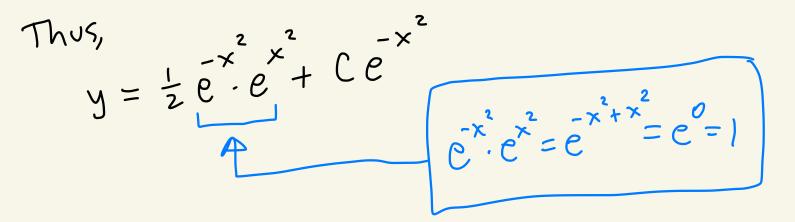
$$\frac{dy}{dx} + 2xy = x$$
Let

$$A(x) = \int 2x \, dx = x^{2}$$

$$A(x) = e^{x^{2}}$$
to get
multiply both sides by $e^{A(x)} = e^{x^{2}}$

A(x) =
$$\int 2x^{2} x^{2} e^{A(x)} = e^{A(x)} = e^{-x}$$
 to get
 $e^{x^{2}} \cdot \frac{dy}{dx} + 2xe^{x^{2}}y = xe^{x^{2}}$
This gives
 $(e^{x^{2}} \cdot y)' = xe^{x^{2}}$
 $(e^{A(x)}y)'$
 $(e^{A(x)}y)'$



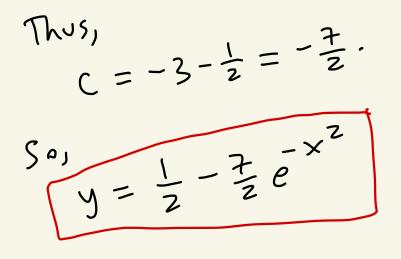


So,

$$y = \frac{1}{2} + Ce^{-x^2}$$

We want $y(o) = -3$. Plugging this into
We above we get
the above $we get$
 $-3 = y(o) = \frac{1}{2} + Ce^{(-)^2}$

$$s_{0,-3} = \frac{1}{2} + Ce^{2} = \frac{1}{2} + C$$



We want to raise

$$xy' + y = 2x$$
, $y(1) = 0$
In $I = (0, \infty)$
First put the equation into a standardized
form by dividing through by x to get
 $y' + \frac{1}{x}y = 2$
Let
 $A(x) = \int \frac{1}{x} dx = \ln|x| = \ln(x)$
 $A(x) = \int \frac{1}{x} dx = \ln|x| = \ln(x)$
 $T = (0, \infty)$
multiply both sides by
 $e^{A(x)} = e^{\ln(x)} = x$
to get
 $xy' + y = 2x$
This gives
 $(x \cdot y)' = 2x$
Integrating with respect to x gives
 $x \cdot y = \int 2x dx$

$$\begin{array}{c} S_{P,y} \\ x \cdot y = x^2 + C \end{array}$$

Thus,

$$y = x + \frac{C}{x}$$

We want $y(1) = 0$. Plugging this
in gives
 $0 = y(1) = 1 + \frac{C}{1}$

$$\begin{array}{c} So, \\ 0 = 1 + C \end{array}$$

Thus,

$$C = -1$$
.
Therefore, the solution is
 $y = x - \frac{1}{x}$

(5) We saw in the previous problems
that the general solution to

$$(x^{2}+9) \frac{dy}{dx} + Xy = 0$$

on $I = (-\omega_{0}, \infty)$ is
 $y = \frac{C}{\sqrt{x^{2}+9}}$
We want the solution to also satisfy $y(0|=1)$.
We want the solution to also satisfy $y(0|=1)$.
Ne want the solution to get
 $|=y(0) = \frac{C}{\sqrt{0^{2}+9}}$
So, $|=\frac{C}{2}$.
Thus, $C = 3$.
So the solution is
 $y = \frac{3}{\sqrt{x^{2}+9}}$